

A GROUP OF FRIENDS

META: TWO HEADED MONSTER



Nine monsters (Abby, Brianna, Chris, Dan, Emily, Fernando, Greg, Hannah and Isabella) have gathered on their computers, all visiting their favorite website `groupnames.org`, which you should visit too. Each has a favorite integer and a favorite group. See if you can identify all of these small groups!

For clarity, we use a lower case variable for each monster's favorite number and an upper case variable for their favorite group. So A is Abby's favorite group and a is her favorite number.

1. All favorite numbers are distinct and all groups are non-isomorphic with distinct orders, except that three have the same order, e .
2. Five groups have the same order as a symmetric group S_n .
3. When sorted alphabetically, the monsters' groups are in increasing order of size.
4. A and F are simple, and A is abelian.
5. B is a maximal subgroup of F and C is a maximal subgroup of E .
6. C has i automorphisms.
7. D is a subgroup of G .
8. E is a 2-Sylow subgroup of $\text{Aut}(A \times A)$.
9. G is the commutator subgroup of the automorphism group $\text{Aut}(A \times A)$.
10. H is the direct product of A and D and has c conjugacy classes.
11. I has b conjugacy classes and h elements of order d .
12. a is the order of F .
13. b is the product of the number of conjugacy classes of F and the number of conjugacy classes of G .
14. d is the number of normal subgroups of F , and is smaller than the order of anyone's group.
15. f is the degree of the smallest faithful transitive permutation representation of H .
16. g is the number of groups of order 1024.
17. h is the number of isomorphism classes of finite groups that have the same order as F .
18. $|E| < f < i < 2|E|$.