



A GROUP OF FRIENDS

META: TWO HEADED MONSTER

Nine monsters (Abby, Brianna, Chris, Dan, Emily, Fernando, Greg, Hannah and Isabella) have gathered on their computers, all visiting their favorite website groupnames.org, which you should visit too. Each has a favorite integer and a favorite group. See if you can identify all of these small groups! For clarity, we use a lower case variable for each monster's favorite number and an upper case variable for their favorite group. So A is Abby's favorite group and a is her favorite number.

1. All favorite numbers are distinct and all groups are non-isomorphic with distinct orders, except that three have the same order, e .
2. Five groups have the same order as a symmetric group S_n .
3. When sorted alphabetically, the monsters' groups are in increasing order of size.
4. A and F are simple, and A is abelian.
5. B is a maximal subgroup of F and C is a maximal subgroup of E .
6. C has i automorphisms.
7. D is a subgroup of G .
8. E is a 2-Sylow subgroup of $\text{Aut}(A \times A)$.
9. G is the commutator subgroup of the automorphism group $\text{Aut}(A \times A)$.
10. H is the direct product of A and D and has c conjugacy classes.
11. I has b conjugacy classes and h elements of order d .
12. a is the order of F .
13. b is the product of the number of conjugacy classes of F and the number of conjugacy classes of G .
14. d is the number of normal subgroups of F , and is smaller than the order of anyone's group.
15. f is the degree of the smallest faithful transitive permutation representation of H .
16. g is the number of groups of order 1024.
17. h is the number of isomorphism classes of finite groups that have the same order as F .
18. $|E| < f < i < 2|E|$.